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Is the North Atlantic Oscillation just a pink noise?

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Abstract

In this paper the authors address the problem of predictability for the NAO index series. The spectral analysis, completed with a bootstrap procedure, shows a rather featureless structure of the index. In other words, the actual time series could be a realisation of many different stochastic processes. An analysis of the Hurst exponent does suggest a slightly red noise as a model for the index, which is interpreted as the NAO being driven by meteorological noise. A nonlinear study of the series (embedding dimension, fractal correlation dimension and leading Lyapunov exponent) shows little predictive performance as well.

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Keywords: NAO; Predictability; Power spectrum; Bootstrap; Hurst exponent; Meteorological noise

1. Introduction

There exists a growing concern on the importance of climate studies in many fields, ranging from purely scientific ones to serious problems in environmental and economic analyses. Understanding climate is a formidable task due to the complexity of the physical system itself and when it comes to predicting, climate scientists are under pressure of governments, industry representatives, news agencies, and conservationists.

Climate is a general idea without a clear-cut definition. Loosely speaking, it is an "average state of meteorological weather" as described by some selected variables, usually air temperature or atmospheric pressure, computed over an adequately large

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area and along a reasonably long time span. In formulas, let Ω be a spatial domain and I a time interval, and let V_m be a meteorological field defined on $\Omega \times I$. Then the corresponding climatic variable V_c is defined [1] by the following formula, where μ is an adequate measure of the spatio-temporal span:

$$V_c = \frac{1}{\mu(\Omega \times I)} \int_{\Omega \times I} V_m \, \mathrm{d}\Omega \, \mathrm{d}t \; . \tag{1}$$

The spatial domain must be always specified, unless it is clear enough from the context. A combination of latitude, ocean and land masses and patterns of atmospheric circulation is the rule for defining climatic areas. The idea of a "reasonably long time interval" is rather diffuse as well. From the historical viewpoint there exists the natural time scale of an average human life, so time intervals of a few tens of years are usually employed. In any case, by shifting the time interval along the time axis a moving average is obtained whose variation is much slower than that of the original meteorological variable.

From the above ideas it follows that the interesting question to think about is "what is climate variability?", rather than "what is climate?". It is known, and well documented from fossil records, that climatic variations have existed both in the time and space domains, and scientifically speaking we turn to "what are the causes of climatic variability?". If some sensible answer is found, then it will be possible to draw conclusions or to hypothesise about the current existence of climate changes.

Several causes are readily identified. First, there exist astronomical phenomena acting on enormous time scales, such as eccentricity changes in the Earth's orbit, variations in the ecliptic angle, and equinox precession. All of them represent variations in the geometric relationship between the Earth and the Sun according to the Milankovich cycles with a scale of 10^4 years. These cycles are inferred from, and used in the study of stratigraphic and palaeontologic records. As regards the air temperature near the ground, the amplitude of the Milankovich wave is of only a few centigrade degrees.

There are shorter cycles with even smaller thermal amplitudes in the range of 10^2 years, which are supposed to be related with volcanic activity episodes, small perturbations of the Earth–Moon system, and solar spots cycles. It is important to emphasise on these ideas, because the available time series with at best 200 data show fluctuations that are quite difficult to tell apart from the longer cycles: This is why making taxative assertions about climate changes is a risky business [2–5].

When dealing with climate and everyday life, the most evident variability source are the yearly seasons, of astronomical origin as well. Most of the apparent "climate changes" are observed and recorded as perturbations of the seasonal cycle, e.g. under the form of unusually warm winters or rainier summers. Some repetitive patterns, like NAO or ENSO, can be observed to oscillate in time scales smaller than the climatic one, say every few years. These waves ride on top of the longer climatic waves, and if the observational window is not large enough, local effects could be taken as general trends, as explained in the above paragraph.

2. Some facts about the NAO

The North Atlantic Oscillation (NAO) is a spatio-temporal climatic pattern where two relationships between the winter weather in distant places of the Northern Hemisphere are found. A first one, rather well known and documented, is detected between the Northernmost Atlantic Ocean area near Iceland, and Scandinavia. The second one has been more recently described to exist between the Southeastern US and the Middle East. Globally considered, they are reminiscent of a spherical vibration pattern with wave numbers 3 or 4 both in latitude and longitude.

From the meteorological standpoint the patterns are driven by differences in the sea surface temperature (SST) [6] at two action centres in the North Atlantic, a first one located at the Açores High in the South, and the second one at the Icelandic Low in the North. Quite often the difference of sea level pressure (SLP) is used instead of the SST difference. SST or SLP are measured at Stykkisholmur in Iceland for the northern end of the NAO and at Açores, Lisbon, or Gibraltar for the southern end. There exist reliable series of averaged annual data reconstructed from several types of records and extending some 150 years into the near past. For each year the difference is computed after careful analyses in order to build the so-called NAO Index. It is a once-a-year feature computed after measurements recorded during the northern hemisphere winter months. Indeed, data do exist at smaller timescales such as monthly records, but the usual yearly index was chosen for this study because the NAO signal in summer months is irrelevant. The NAO is in positive (resp. negative) phase when the index is positive (resp. negative). Throughout this paper, the index series of SST anomalies (140 data), available at www.jisao.washington.edu/data_sets/nao/ will be used. The data was checked in order to see whether it had some trend. Polynomials of first, second and third degree were fitted, but the r^2 statistic, of a measure of the percentage of data really well explained by the fitted curve, yielded the non-significant values 0.037, 0.051 and 0.132, so the data was directly employed. See Fig. 1.

The NAO index is compared with many other variables, either simple or composite ones, in order to discover hidden relationships and correlations leading to a deeper

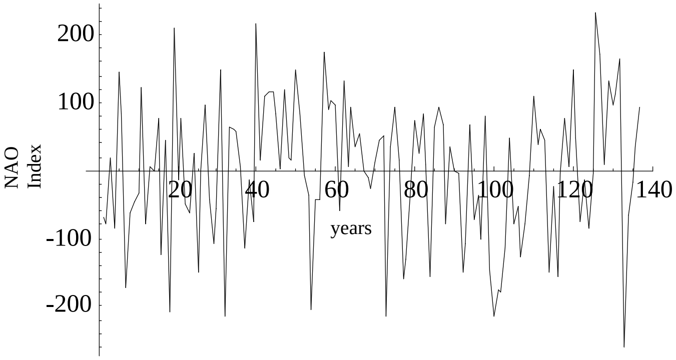

Fig. 1. Time series of the NAO index (arbitrary units).

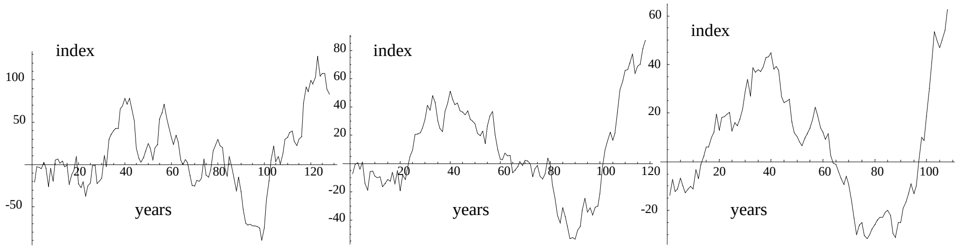

Fig. 2. From left to right: 10, 20, and 30 point moving averages of the NAO index.

understanding of the NAO and of the general climate system. As the available series is a rather short one, the results of any typical predictability analyses must be taken with caution [7]. This should foster the efforts in physically understanding the tetrapolar spatial structure of the oscillation.

To start with, several standard moving average (simple arithmetical means) smoothing of the data yielded the results shown in Fig. 2.

An oscillatory pattern with a period over 20 years is very apparent, in line with rather frequent phrases like "...a last period of some 25 years has been observed where the NAO appears to be in the positive phase..." (see e.g. [8]) followed by comments on the possible anthropic influences on climate evolution.

Fig. 3 shows the periodogram of the index series. The frequencies ω(cycles/year) in the horizontal axis are defined by the formulae

$$\omega = \frac{2\pi j}{n}, \quad j \leqslant \left[\frac{n}{2}\right] , \tag{2}$$

where $[n/2]$ is the integer part of $n/2$. The last value of ω, the Nyquist frequency, is $\omega_N = \pi$.

Let us consider the most prominent peaks in the periodogram by naming them A–K. Peak C is the signal of a 9.5 year cycle, close to the solar spots one, and it was taken as a reference by disregarding all other peaks lower than it. Peak D, the highest one, is associated with a 7.5 year cycle. It should be considered jointly with peaks E (5.6 years), F (3.7 years) and G (3.2) as marking a possible semidecadal cycle. Peak B is associated to a 22.1 year cycle, thus confirming the results of the elementary smoothing performed previously, as well as the observations quoted before. Peak A marks a 70.7 year cycle with no apparent interpretation in such a short series. Finally, the group H, I, J, K (cycles: 2.7, 2.4, 2.2, and 2.1 years) is readily understood as a quasi-biennial oscillation (QBO).

Nevertheless, the general appearance of the periodogram is rather "fat" and this is why noise must be taken into account when dealing with the NAO. Before proceeding into a noisy analysis, the spectral analysis was completed. According to Priestley [9], the periodogram does not provide a good estimation of the process structure, and accordingly the power spectrum was computed by smoothing the periodogram. The general trend of the power spectrum indicates that most of the structure is concentrated

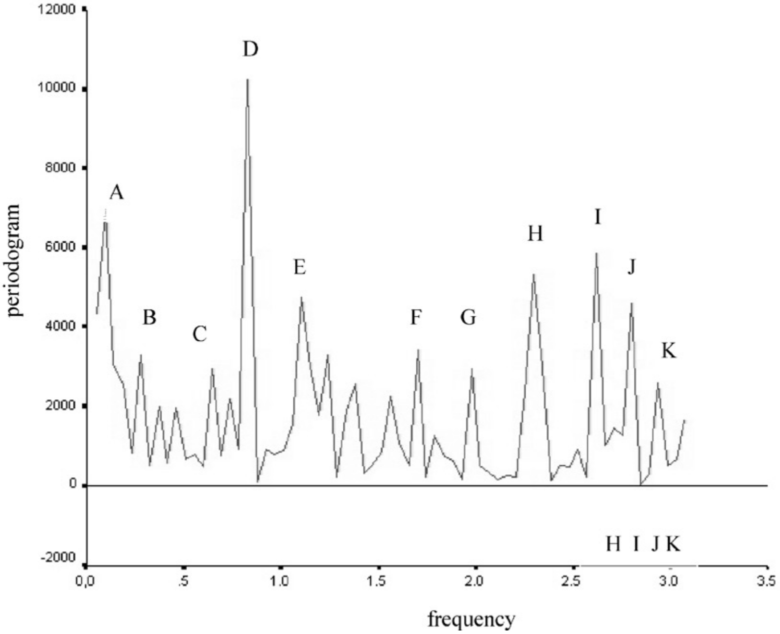

Fig. 3. Periodogram of the NAO index series (arbitrary units).

on the low and middle-range frequencies, while the QBO signal has a significantly smaller contribution.

A bootstrap analysis was performed, trying to assess the reliability of this model behaviour. Bootstrapping is a statistical tool for the study of features of data sets when the total population is not large enough to be confident on the usual "sampling and hypothesis testing" technique.

The rationale behind the bootstrap is to take a large number of equal samples with replacement out of the population and to compute the average over them of the sample statistic one is interested in. After that, the theory shows that usually these averages do converge to the corresponding population statistic when the sample size is increased. There exist bootstrap procedures for most of the current statistics or interesting features (see a readable general account in Ref. [10], and the specific case of the power spectrum in Ref. [11]). In this case the resampling procedure and computation of the spectrum was repeated 1000 times for every sample size. The performance of the bootstrap statistic is usually presented by means of confidence intervals or strips, as shown in Fig. 4 for the power spectrum of the NAO index.

Fig. 4 shows the 95% confidence strip for the power spectrum. The trumpet-shaped ends of the strip are due to the influence of boundary values. In any case, the strip is wide enough to allow many very different power spectra as reasonable candidates for the actual structure of the NAO signal; only the quasi-decadal components are more precisely described. Therefore, the noisy hypothesis must again be considered, and

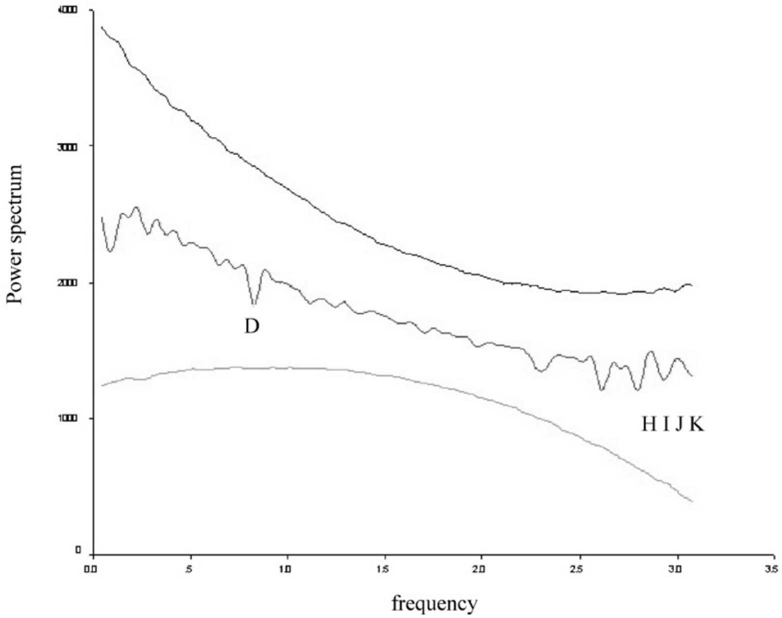

Fig. 4. Power spectrum of the NAO signal obtained by smoothing the periodogram (arbitrary units), and Bootstrap confidence band for the power spectrum.

every effort must be done in Physics in order to assess noise to possible tropospheric causes or to admit stratospheric activity as well. Both show a much faster variability than the climatic scale and are natural candidates in the construction of noisy models.

3. Predictability analyses

In this paper only prediction in the time variable will be considered, although it has been pointed out that spatial aspects of the NAO should be emphasised. The most general form for a predictor is

$$\hat{X}_t = F(\{X_\tau\}_{\tau<t}) . \tag{3}$$

The simplest one is known as permanence (or persistence) and it amounts to saying that the next expected value is just the last observed one:

$$\hat{X}_t = X_{t-1} . \tag{4}$$

This technique is rather useful in the short-term range, and defeating permanence is a real challenge in actual practice. Moreover, separating general trends from permanence is an interesting subject in itself [2]. Nevertheless, permanence does not yield good results beyond a few time units. In fact, it is based on the idea that all over the series, the averaged difference

$$\langle |X_{t+1} - X_t| \rangle \tag{5}$$

is small. In the general case, the averaged difference with a lag of k time units must be considered as a function of k. As a rule, it scales as

$$\langle |X_{t+k} - X_t| \rangle \approx k^H , \tag{6}$$

where the exponent H is called the Hurst exponent. In the case of a hypothetical multi-self-affine behaviour, the local exponent $H = h(t)$ is called the Hölder exponent. Mathematically, H can be considered as a measure of "irregularity" in the signal. For continuous processes, $H < 1$ means that the signal cannot be represented by a differentiable function (indeed, for a discrete process represented by line segments joining the discrete values this is immediately true) and the smaller H, the more irregular the signal behaviour. The case $H = \frac{1}{2}$ corresponds to a white noise Brownian signal.

In physical terms, H measures the residual memory of the time series. Actual computations of H include estimating the growth of the deviation and averaging on successive intervals. The value $H = \frac{1}{2}$ of the white noise is a limiting case: If $H > \frac{1}{2}$, the signal is called red, and blue if $H < \frac{1}{2}$. More regularity (red) than the white noise signal implies more memory of past values, i.e., permanence, while less regularity (blue) will be called antipermanence.

Computation of H for the NAO index series yields a value slightly larger than the critical value $H = \frac{1}{2}$ of the white noise case:

$$H_{\mathrm{NAO}} = 0.6473 \tag{7}$$

and this means a certain redness (maybe "pinkness" would be a suitable term) or memory of past episodes in the NAO signal. This value is practically the same one obtained by Koscielny-Bunde et al. [12,13] as the exponent of a universal power law governing atmospheric variability, and the result shown here could be interpreted as further evidence of the universality of the law.

It is hard to plainly assume the NAO to be an almost unstructured noisy signal, so as a check a nonlinear analysis (see e.g. [14]) was performed. It must be stressed that the series is so short that the results are only tentative ones. The main parameters used in nonlinear analysis are: The embedding dimension, the fractal dimension, and the leading Lyapunov exponent.

The embedding dimension is an integer d which is the dimension of a metric vector space where the series is reconstructed as a set of vectors

$$V_d = \{(X_j, X_{j-1}, \ldots, X_{j-d+1}), \quad j = 1, 2, \ldots, n\} \tag{8}$$

and it is computed through the false nearest neighbours (FNN) technique. Two vectors in V_d are called FNN if the distance between them is small in V_d but becomes larger when they are upgraded to V_{d+1} by addition of a term to each vector. As a rule, the percentage of FNN settles to 0 after a number of upgradings that defines the embedding dimension. In this case, it was found that $d = 6$.

The fractal dimension is a positive real number $f \leqslant d$ measuring the spatial complexity of the set of points (or vectors) V_d in d-dimensional space. Actually, the fractal dimension is only properly defined for infinite self-similar sets obtained as a result of some recursive procedure. In the case of finite data sets only a pseudo-fractal dimension (correlation dimension) can be computed. This number is calculated by counting the number of points of V_d inside a set of balls with variable radii centred at the points of the set and observing its evolution as the radii increase. For the NAO case, it was found that $f = 4.58 \pm 0.31$. This value range indicates a complicated structure and little deterministic structure in the data.

The leading Lyapunov exponent L is a way of representing how fast the orbits of two nearby initial conditions in a dynamical system separate from each other as time evolves, in other words, it measures to what extent the initial conditions determine future values. In this case the dynamics is given by the succession of the embedded vectors. L is related with the predictability limit of the series. It is computed by observing the distance between homologous data along two initially close orbits and averaging it after some integration steps. It was found that for the NAO index L was given by

$$\begin{aligned} L &= 0.19 \pm 0.08 \text{ (with 4 integration steps)} \\ &= 0.13 \pm 0.065 \text{ (5 steps)} \\ &= 0.102 \pm 0.07 \text{ (6 steps)} \end{aligned}$$

The prediction horizon or predictability limit is defined as the inverse of L. The variability in the value of L is around 50% of the estimated value—and increases with the number of integration steps. Therefore, by accepting the 5 step computation a prediction horizon is found ranging between 5.13 and 15.38 years. These values suggest again the quasi-decadal features observed in the NAO signal [15]. In other words, the NAO index shows a rather small predictability, thus favouring the pink noise option.

4. Conclusions and comments

The NAO index is a rather short series and the results of time series analyses must be very carefully considered. Indeed, a study like Ohira et al. [16] on very long series of currency change rates would be impossible. A classical time series analysis looking for periodic components was complemented by the bootstrap technique applied to the power spectrum, yielding a wide 95% confidence band. This shows that there are many candidates to the underlying stochastic process, thus yielding a very small predictability for the index, most of it concentrated in the middle (or semidecadal range) range. A tentative interpretation points out to the permanence of some tropospheric or stratospheric signal on ranges longer than their usual time scales. The problem of permanence over different time scales has been addressed in Bunde et al., [2] by checking the outputs of a set of coupled atmosphere–ocean general circulation models.

From the noisy viewpoint, the NAO index appears to be a pink signal, to which a slight predictive capability can be assessed. Again, this should be an extrapolation of meteorological predictability, either at the tropospheric or stratospheric levels. Maybe this could be paraphrased with the words of Jonathan Swift [17]:

> "...having prepared all their musical instruments, played on them for three hours without intermission; so that I was quite stunned with the noise; neither could I possibly guess the meaning, till my tutor informed me". (A voyage to Laputa, Chapter 2)

An open problem described in Von Storch et al., [18] is to discriminate whether noise is a *cause* or an *effect*. The complexity of the climate system is so high that even the more sophisticated models cannot account for every possible variability source: Only some of them are considered essential, while the rest are parameterised or added as noise. If the study of model outputs yields coherent answers when forced with noises, then it could be assessed that in some sense the NAO is a climatic pattern driven by meteorological noises. Nevertheless the reverse idea, i.e., that the NAO is a kind of "conveyor belt" for meteorological variables, could be also true [19,20] and should be explored. Indeed, more evidence is needed.

The Coupled Model Intercomparison Project (CMIP, see Stephenson [21]) has studied the outputs of 17 coupled atmosphere–ocean general circulation models of the above type. According to it, at least 13 of the 17 competing models do reproduce the tetrapolar spatial structure of the NAO, and a fictive NAO index has been computed after averaging the results of not less than 10 models. This artificial index is a slightly red noisy signal, in accordance with the analysis in this paper.

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