Proof Without Words: Limit of a Recursive Arithmetic Mean
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Define the sequence \((a_n)_{n \geq 1}\) by \(a_{n+1} = (a_n + a_{n-1})/2\) for \(n \geq 2\) with two positive initial values \(a_1\) and \(a_2\). Then

\[
\lim_{n \to \infty} a_n = \frac{a_1 + 2a_2}{3}.
\]

Proof.

Summary. Visual proof of the limit of a recursive arithmetic mean sequence.

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