Consider the set, $S$ of polynomials in one variable over the integers with zero coefficient on the linear term. That is to say consider:

$$S = \{a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + a_{n-2} \cdot x^{n-2} + \cdots + a_2 \cdot x^2 + a_0\}$$

Now it’s easy to verify that $S$ is an integral domain. But the delightful surprise is that this integral domain does not have unique factorization into irreducibles (nonunit elements $x$ such that if $x = yz$ then $y$ or $z$ is a unit) and that this is clear immediately from what follows. Consider $x^6$. This can be written as $x^2 \cdot x^2 \cdot x^2$ or $x^3 \cdot x^3$. Both $x^2$ and $x^3$ are clearly irreducible in $S$. And since these two factorizations into irreducibles contain different numbers of factors, they are distinct.

As a bonus we also find that $x^2$ and $x^3$ are not prime illustrating that prime and irreducible are separate concepts. For example, $x^2$ divides $x^3 \cdot x^3$ while not dividing either factor.

REFERENCE


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**Proof Without Words:**

**Alternating Sum of an Even Number of Triangular Numbers**

ÁNGEL PLAZA
ULPGC
35017-Las Palmas G.C., Spain
aplaza@dmat.ulpgc.es

$$t_k = 1 + 2 + \ldots + k \Rightarrow \sum_{k=1}^{2n} (-1)^k t_k = 2t_n$$

E.g., $n = 3$:

$$-t_1 + t_2 - t_3 + t_4 - t_5 + t_6 = 2t_3$$