Proof Without Words: Alternating Row Sums in Pascal’s Triangle

ÁNGEL PLAZA
Universidad de Las Palmas de Gran Canaria
Las Palmas, Canaria, Spain
angel.plaza@ulpgc.es

Theorem. For any integers $0 \leq j \leq m \leq n$,

$$
\sum_{k=j}^{m} (-1)^k \binom{n}{k} = (-1)^j \binom{n-1}{j-1} + (-1)^m \binom{n-1}{m}.
$$

and in particular if $j = 0$ and $m = n$, then

$$
\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0
$$

(by defining as usual $\binom{n}{-1} = 0 = \binom{n-1}{n}$).

Proof. For simplicity we show the case when $j$ is even; the odd cases can be obtained by reversing the role of $+$ and $-$.

Summary. Based on the Pascal’s identity, we visually demonstrate that the alternating sum of consecutive binomial coefficients in a row of Pascal’s triangle is determined by two binomial coefficients from the previous row.

ÁNGEL PLAZA (MR Author ID: 350023) received his masters degree from Universidad Complutense de Madrid in 1984 and his Ph.D. from Universidad de Las Palmas de Gran Canaria in 1993, where he is a full professor in applied mathematics. He is interested in mesh generation and refinement, combinatorics, and visualiza-